

Please check the examination details below before entering your candidate information

Candidate surname					Other names				
Centre Number					Candidate Number				

Pearson Edexcel Level 3 GCE

Friday 19 May 2023

Afternoon

Paper reference **8FM0/26**

Further Mathematics
Advanced Subsidiary
Further Mathematics options
26: Further Mechanics 2
(Part of option J)

You must have:
 Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8 \text{ m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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1. Three particles of masses $4m$, $2m$ and km are placed at the points with coordinates $(-3, -1)$, $(6, 1)$ and $(-1, 5)$ respectively.

Given that the centre of mass of the three particles is at the point with coordinates $(\bar{x}, \bar{y})$

(a) show that $\bar{x} = \frac{-k}{k+6}$ (3)

(b) find $\bar{y}$ in terms of k . (2)

Given that the centre of mass of the three particles lies on the line with equation $y = 2x + 3$

(c) find the value of k . (2)

A fourth particle is placed at the point with coordinates $(\lambda, 4)$.

Given that the centre of mass of the four particles also lies on the line with equation $y = 2x + 3$

(d) find the value of λ . (2)

centre of mass
↓

(a) Mass	$4m$	$2m$	km	$(6+k)m$
Distance ↑ from y-axis	-3	6	-1	$\bar{x}$

Taking moments about the y-axis: ①

$$(6+k)m\bar{x} = -3(4m) + 6(2m) - 1(km) \quad ①$$

$$(6+k)\bar{x} = -12 + 12 - k$$

$$\bar{x} = \frac{-k}{k+6} \quad ①$$


Question 1 continued

(b) mass	4m	2m	km	$(b+k)m$	
Distance	-1	1	5	$\bar{y}$	← from x-axis

Taking moments about y-axis:

$$(b+k)m\bar{y} = -1(4m) + 1(2m) + 5(km) \quad (1)$$

$$(b+k)\bar{y} = -4 + 2 + 5k$$

$$\bar{y} = \frac{5k-2}{b+k} \quad (1)$$

$$(c) \quad \bar{y} = 2\bar{x} + 3$$

$$\frac{5k-2}{b+k} = \frac{-2k}{b+k} + 3 \quad (1) \quad \leftarrow \text{use results from (a) and (b).}$$

$$5k-2 = -2k + 3(b+k)$$

$$4k = 20$$

$$k = 5 \quad (1)$$

(d) C.o.m stays on the line, so $(1,4)$ must lie on that line.

$$y = 2x + 3$$

$$4 = 2\lambda + 3 \quad (1)$$

$$\lambda = \frac{1}{2} \quad (1)$$



Question 1 continued

Lined area for writing the answer to Question 1.

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Question 1 continued

Lined area for writing answers.

(Total for Question 1 is 9 marks)



2. A particle P is moving along the x -axis.

At time t seconds, $t \geq 0$, P has acceleration $a \text{ ms}^{-2}$ and velocity $v \text{ ms}^{-1}$ in the direction of x increasing, where

$$v = e^{2t} + 6e^t - kt$$

and k is a positive constant.

When $t = \ln 2$, $a = 0$

- (a) Find the value of k .

(4)

When $t = 0$, the particle passes through the fixed point A .

When $t = \ln 2$, the particle is d metres from A .

- (b) Showing all stages of your working, find the value of d correct to 2 significant figures.

[Solutions relying entirely on calculator technology are not acceptable.]

(4)

$$(a) \quad a = \frac{dv}{dt} \quad (1)$$

$$a = 2e^{2t} + 6e^t - k \quad (1) \quad \leftarrow \frac{d}{dt} e^{at} = ae^{at}$$

$$\text{When } t = \ln 2, a = 0. \quad 0 = 2e^{2\ln 2} + 6e^{\ln 2} - k \quad (1)$$

$$0 = 8 + 12 - k$$

$$\therefore k = 20 \quad (1)$$

$$(b) \quad v = \frac{dx}{dt} \Rightarrow x = \int v \, dt \quad (1)$$

$$x = \int_0^{\ln 2} e^{2t} + 6e^t - 20t \, dt$$

← limits come from boundary conditions ($t=0, \ln 2$) given in the question.

$$x = \left[\frac{1}{2} e^{2t} + 6e^t - 10t^2 \right]_0^{\ln 2} \quad (1)$$

$$x = \left[\frac{1}{2} e^{2\ln 2} + 6e^{\ln 2} - 10(\ln 2)^2 \right] - \left[\frac{1}{2} e^{2 \times 0} + 6e^0 - 10(0) \right] \quad (1)$$

$$x = 7.5 - 10(\ln 2)^2$$

$$x = 2.7 \quad (2.s.f.) \quad (1)$$



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Question 2 continued

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(Total for Question 2 is 8 marks)



3. A girl is cycling round a circular track.

The girl and her bicycle have a combined mass of 55 kg.

The coefficient of friction between the track surface and the tyres of the bicycle is μ .

The track is banked at an angle of 15° to the horizontal.

The girl and her bicycle are modelled as a particle moving in a horizontal circle of radius 50 m

The minimum speed at which the girl can cycle round this circle without slipping is 4.5 m s^{-1}

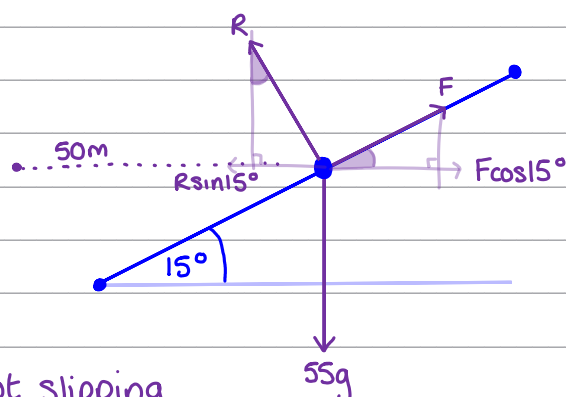
Using the model, find the value of μ .

(9)

Horizontal motion using $F = m \frac{v^2}{r}$ ①

$$R \sin 15 - F \cos 15 = 55 \times \frac{4.5^2}{50} \quad \text{①}$$

$$R \sin 15 - F \cos 15 = 22.275 \quad \text{①}$$



Resolve vertical forces. ① ← she is not slipping so vertical forces are balanced.

$$55g = R \cos 15 + F \sin 15 \quad \text{①}$$

$F = \mu R$

$$55g = R \cos 15 + \mu R \sin 15 \quad \Rightarrow \quad R = \frac{55g}{\cos 15 + \mu \sin 15} \quad \text{①} \quad \text{①}$$

$$R \sin 15 - \mu R \cos 15 = 22.275 \quad \text{②}$$

Substitute ① into ②.

$$\frac{55g \sin 15}{\cos 15 + \mu \sin 15} - \frac{55g \mu \cos 15}{\cos 15 + \mu \sin 15} = 22.275$$

$$55g \sin 15 - 55g \mu \cos 15 = 22.275 \cos 15 + 22.275 \mu \sin 15$$

$$55g \sin 15 - 22.275 \cos 15 = \mu (55g \cos 15 + 22.275 \sin 15) \quad \text{①}$$

$$\mu = \frac{55g \sin 15 - 22.275 \cos 15}{55g \cos 15 + 22.275 \sin 15} = 0.224 \quad \text{①}$$



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Question 3 continued

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Question 3 continued

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Question 3 continued

Lined area for writing answers.

(Total for Question 3 is 9 marks)



4.

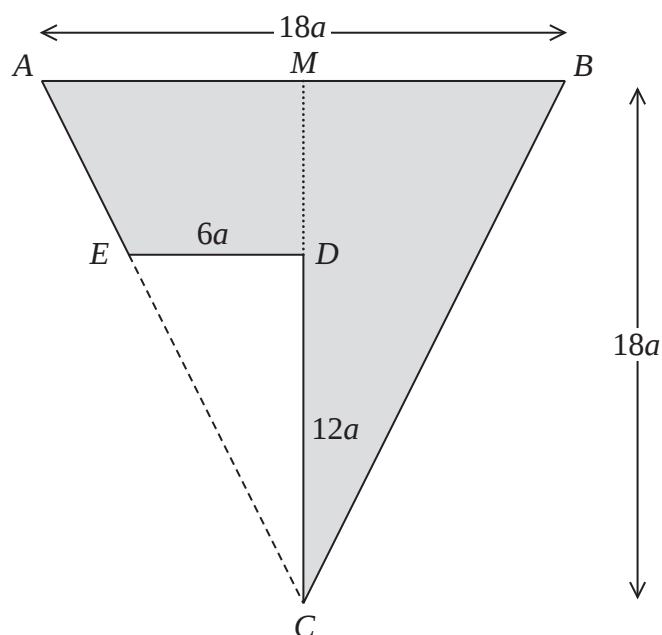


Figure 1

A uniform triangular lamina ABC is isosceles, with $AC = BC$. The midpoint of AB is M . The length of AB is $18a$ and the length of CM is $18a$.

The triangular lamina CDE , with $DE = 6a$ and $CD = 12a$, has ED parallel to AB and MDC is a straight line.

Triangle CDE is removed from triangle ABC to form the lamina L , shown shaded in Figure 1.

The distance of the centre of mass of L from MC is d .

- (a) Show that $d = \frac{4}{7}a$ (4)

The lamina L is suspended by two light inextensible strings. One string is attached to L at A and the other string is attached to L at B .

The lamina hangs in equilibrium in a vertical plane with the strings vertical and AB horizontal.

The weight of L is W

- (b) Find, in terms of W , the tension in the string attached to L at B (3)

The string attached to L at B breaks, so that L is now suspended from A .

When L is hanging in equilibrium in a vertical plane, the angle between AB and the downward vertical through A is θ°

- (c) Find the value of θ (7)

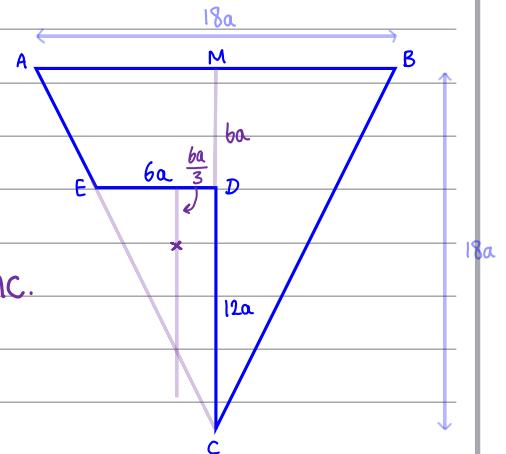


Question 4 continued

(a) Area of $\triangle ABC = \frac{1}{2} \times (18a)^2 = 162a^2$
 C.O.M of $\triangle ABC$ lies on MC.

Area of $\triangle CDE = \frac{1}{2} \times 12a \times 6a = 36a^2$
 C.O.M of $\triangle CDE$ is $\frac{2}{3}$ along DE $\Rightarrow \frac{6a}{3}$ from MC.

Area of L = $162a^2 - 36a^2 = 126a^2$
 C.O.M of L is d from MC



Moments about the line MC ①

this c.o.m will $\rightarrow 126a^2d = 162a^2 \times 0 + 36a^2 \times \frac{6a}{3}$ ① ← this c.o.m would be on the right of mc
 be on the right of mc $126a^2d = 36a^2 \times 2a$ ①
 of mc

$$126a^2d = 64a^3$$

$$126d = 64a$$

$$d = \frac{64a}{126}$$

$$d = \frac{4}{7}a$$
 ①

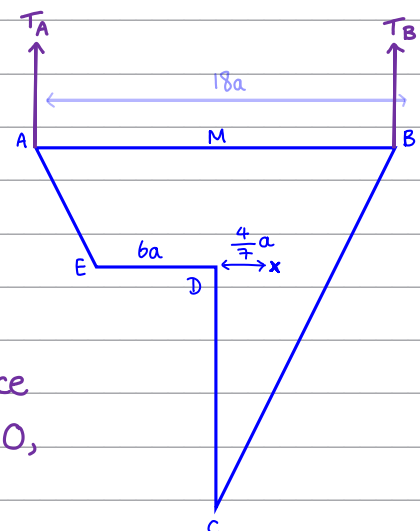
(b) Moments about A: ①

$$18a \times T_B = W \times \left(\frac{18}{2}a + \frac{4}{7}a \right)$$
 ①

$$T_B = \frac{67}{7}aw \div 18a$$

$$T_B = \frac{67}{126}W$$
 ①

↑
 horizontal distance
 of T_A from A is 0,
 so ignore T_A .



Question 4 continued

(c) Moments about AB ^① using model from (a) to find $\bar{y}$

$$12ba^2\bar{y} = 162a^2 \times ba - 36a^2 \times (ba + 4a) \quad \text{①}$$

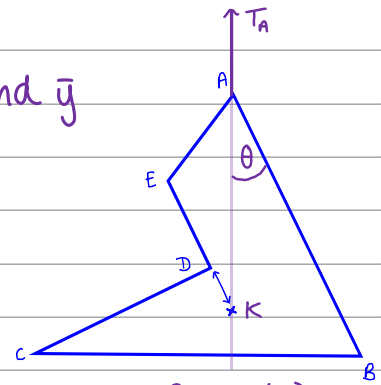
↑
L

↑

ΔABC C.O.M
is $\frac{2}{3}$ from apex
($18a \div 3 = 6$)

↑

ΔCDE C.O.M is
 $\frac{2}{3}$ from apex
($12a \div 3 = 4a$)



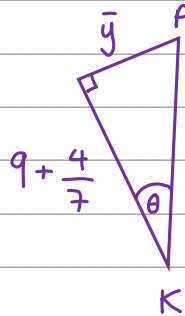
C.O.M (K) is vertically below T_A , making AK a straight line.

$$12ba^2\bar{y} = 972a^3 - 360a^3$$

$$126\bar{y} = 612a$$

$$\bar{y} = \frac{34}{7}a \quad \text{①}$$

$$\tan\theta = \frac{\frac{34}{7}a}{\left(9 + \frac{4}{7}a\right)} \quad \text{①}$$



← translated from diagram using known values of $\bar{x}$ and $\bar{y}$

$$\tan\theta = \frac{34}{67} \quad \text{①}$$

$$\theta = 27^\circ \quad \text{①}$$

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Question 4 continued

Lined area for writing the answer to Question 4.



Question 4 continued

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(Total for Question 4 is 14 marks)**TOTAL FOR FURTHER MECHANICS 2 IS 40 MARKS**